Tuning a hurdy gurdy 's tangents

in relation to the experience of pure intervals

for playing alone or unisono

Ernic Kamerich, January 2007, slightly improved Februari 2009

Introduction

On a hurdy gurdy playing alone there is no harmony apart from the combination of melody and drone, so the obvious perfect way for tuning the tangents is by making all intervals pure between chanterelle and bourdon: a natural harmonic scale. That works well and makes the instrument sound sweet and beautiful.

However, most hurdy gurdies can use more than one bourdon and then things get more complicated. Moreover, playing together with other instruments may ask for a different tuning. Here will be explained what can be done without losing the sweet character of natural harmonic tuning. All is explained in detail, but if you just want to try the result, read "A practical proposal" and "A working scheme".

The experience of pure intervals on a hurdy gurdy

Pure tuning is very important for some intervals, less so for others. Most people can (learn to) hear if a fifth, a fourth, a major third and eventually a minor third is pure, but pureness of a second is more difficult to hear.

However, on a hurdy gurdy these intervals are usually added to one or two octaves: if you play a b' on a g' chanterelle (so the third of the lowest note), it may be accompanied by a G and/or g bourdon/mouche/chien, so you don't get a third but a tenth or a seventeenth (2 octaves plus a third). That is rather important for the experience of (im)pureness. For instance, pureness of a ninth, a second plus an octave, can be heard much better than a second. At the contrary, pureness of a minor tenth, a minor third plus an octave, is much less discernible than pureness of a minor third itself. These observations will be discussed and explained later. Where compromises should be taken, these things can be important.

Why not use modern equal temperament?
In modern equal temperament fifths are slightly impure, and you may accept that as tolerable or not. Most players don't like impure fifths. Moreover, tuning in equal temperament yields rather impure thirds. These may sound acceptable for modern ears, but hearing pure thirds for some time makes the ears get used to them and then returning to equal temperament is felt as a serious loss. Many people don't want to bother about tuning and want to play modern equal tuning, possibly with a slight modifications in order to get pure fifths with the bourdons. If you play traditional or old music, you might give a nicer tuning a try.

In the group for renaissance and medieval music where I play, there was much resistance to changing the temperament. But after one serious try to play renaissance music in mean tone temperament, everybody was convinced. It sounds more transparent, more pleasant and old persistent problems with impurities disappeared. Mean tone temperament yields pure major thirds but rather impure fifths, so it is not fine for medieval music and monophonic music with a drone.

For medieval music Pythagorean temperament works; in fact, the medieval treatises on tuning a hurdy gurdy all indicate Pythagorean tuning. A large part of medieval music suitable for hurdy gurdy does not include the third on the bourdon, often it is in dorian or hypodorian mode. On the almost diatonic medieval hurdy gurdy with its range of an octave of a none, the bourdon should be one octave under the note of the first key for dorian mode; for hypodorian mode it should be one octave under the note of the fourth key. So with a c' melody string one would use a drone d or g (and with the standard g' melody string one would use a drone a or d').

When playing together, for instance with a modern instrument, especially when it has a fixed tuning (melodeon, harmonica, piano), it may be wise to make some adaptations in the tuning of the hurdy gurdy.

When playing baroque duets for hurdy gurdy, natural harmonic temperament yields some awkward intervals, especially between the third and the sixt, so this music asks for a different temperament as well.

For most monophonic music with a drone later than about 1450 the natural harmonic scale is ideal.

**Barriers in using natural harmonic scales**

In practice, natural harmonic tuning of the tangents works perfectly if the interval between the bourdon and the open melody string is fixed. However, in most cases there are two or more bourdons available. You might want to play in C and in G on the same chanterelle, for instance. Then you get a conflict already: if the A is tuned as a pure sixth to the C, it proves to be
impure to the G and this impureness can be heard clearly, because it is a second plus one or two octaves. There are choices to be made. In the following such choices are discussed and theoretical and practical help is given to make your own choices.

**A practical proposal**

If you get impatient and don't want to read further on, you might want to see my proposal for a practical tuning system. Here it is for playing with bourdons in C and G. The choice for this system will be explained in the section *The proposed tuning explained* and you can find numerical data on it in the table ("preferred pure temperament").

Tune the G and make G→C, C→F and G→D, D→A pure fifths. Then make C→E, G→B, D→Fis, A→Cis, D→Bes, G→Es and C→As pure thirds/sixths. The result will be that E-B, B-Fis, F-Bes, Bes-Es and Es-As will be pure fifths/fourths as well.

**A working scheme**

Here it is supposed that your hurdy gurdy has a chanterelle in g', two bourdons, possibly in G and c/d and at least one string more in between.

Slacken the pullers of the chiens. Be sure that cottoning, resin on the wheel and the pressure of the strings on the wheel are okay.

Now check the positions of the nut and the bridge (the ends of the vibrating part of the open chanterelle). The tangent of the fourth tangent (b'-key) must yield a pure third, so the distance to the nut should be 1/5 of the length of the open string (possibly 1/5x345mm), about 2 mm shorter than with modern equal temperament. If the position of the tangent can not be changed enough, the position of the nut must be changed so that the tangents of this third and the following tangent (the fourth) can both be accommodated well. The perfect distance of the fourth is 1/4 of the vibrating length.

The tangent of the highest note without deviation must have the correct position. If this tangent should yield the double octave, the distance to the bridge should be 1/4 of the vibrating length of the open chanterelle. This is true for any normal tuning system. If that is not the case (about 1mm deviation me be acceptable), not any tuning system will work for the highest notes. Then the position of the bridge must be changed. If the bridge is glued to the soundboard, possibly you can change its position a little with the wire that pulls the bridge to the tailpiece (after having slackened the chanterelles).
Otherwise, you may better return with it to the maker or consult a violin repair shop.

If you don't have machine tuning pegs or fine tuning adjusters, it is wise to use a tool for easy turning the tuning pegs in order to get precisely what you want. You may test in which direction the tangent is to be moved by pressing the key some more, making the tone a little sharper; listen if it gets better or worse. If you hear a beat (something like vibrato) the tuning is nearly pure: try to get the frequency of this beat to zero. If you are used to modern (equal temperament) tuning, the pure thirds will be considerable flat in your ears. Lower the thirds more than you think until you hear a quiet sonorous harmony without beats.

1. Tune the chanterelle to g', tune a bourdon to G (pure to the chanterelle) and tune the tangents of g" and g''' (if available) pure to the bourdon G.

2. Tune b' and b" pure to the bourdon. Remember that thirds (the pure b's) are very flat in modern ears.

3. Switch off that G bourdon, use the open g' chanterelle to tune a string to c, c' or c'', the highest possible. Tune the tangents of e" and e''' to the c, again very flat.

4. Then tune the tangents of f" and f''' (more difficult!) If you are not content, you may switch off the chanterelle, switch on the G bourdon and tune this to F in relation to that c-string, switch on the chanterelle again, check it to the c-string and tune the f' and f" tangents to this F bourdon.

5. Tune again the chanterelle to g', a bourdon in G, pure to the chanterelle and check the tangents of g" and g''', b' and b". Now tune the tangents of d" and d''' pure to the G.

6. Tune another string (bourdon/chien/mouche) to d or d' as a pure fifth to the G bourdon and check this against the tangents of d" and d''' . Switch off the G bourdon and tune the tangents of a' and a" to the d. Then tune the tangents of fis" and fis''' as pure thirds to that d-string.

You are ready with the critical part. Tuning cis, bes, es and as according to the pure system above is much more difficult. However, don't bother too much: if you can not hear the pureness very well, who cares? You may even dislike the melodic distances of these notes or the differences with the tuning of other instruments if you play together. Then you can change the tuning of these four notes a little: probably your ears will not be offended by the less perfect tuning of these notes to the bourdons. So these other tangents may be left unchanged or you may tune them in equal temperature with an electronic tuner or an other instrument or use a high string of the hurdy gurdy as a reference by tuning it to what you need.
When you have tuned the tangents of the first chanterelle, you may tune the tangents of the other chanterelle(s) simply by tuning them to the already tuned chanterelle. That is easy if they are tuned the same or if the distance is a fifth, an octave or a twelfth (fifth+octave). If the usual distance is a fourth, you may better lower the lowest string one tone for the tuning process. It is essential that the tangents touch the string simultaneously, as accurately as possible. An easy trick to test for the direction in which the tangent is to be moved: pull one of the chanterelles a little with your fingers at the pegs-site of the tangents: the tone will be a little sharper and you can hear if it gets better or worse.

**Hearing impurities explained**

For fifths, fourths, thirds and sixths it is easy to hear if they are pure or not. Even the pureness of the ninth can be discerned easily, better than the second, in fact. The pureness of the major tenth, third plus one octave, is very easy to hear. The minor tenth, a minor third plus an octave, is rather unclear and even a little tricky. Why all these effects? That will be explained in the following.

Each musical tone can be considered as a combination of a set of (sinusoidal) tones, each of which has a frequency that is a multiple of the ground frequency. If you hear a tone of 500 Hz, usually it will contain tones of 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, ... Hz. These are called the **harmonics** of that 500 Hz tone. The harmonics are numbered 1,2,3,..., where number 1 is the lowest, the ground tone.

The harmonics of a tone are (especially given here as an example for ground tone c)

1. the tone itself (c)
2. octave (c')
3. octave plus fifth (g')
4. second octave (c'')
5. two octaves plus third (e'')
6. two octaves plus fifth (g'')
7. a strange note, well known from natural trumpets, 1/3 semitone lower than two octaves plus a minor seventh (a little lower than bes'')
8. third octave (c''')
9. three octaves plus a second (d'''
10. three octaves plus a third (e''')
11. again a strange note, 1/2 semitone higher than three octaves plus a fourth, so not corresponding to a regular interval in the twelve-tone-system
12. three octaves plus a fifth (g'''')
13. ......

If you hear a tone of 601 Hz, it will deliver as well 601, 1202, 1803, 2404, 3005, 3606, ... Hz. Let's suppose that this tone is combined with that previous tone of 500 Hz. Now you see that two of their harmonics are rather near to each other: 3000 (6th harmonic of 500 Hz) and 3005 Hz (5th harmonic of 601 Hz). These two will make the sound vibrate to the difference 5Hz and this gives a dirty and unpleasant effect. If the second ground tone is changed to 600 Hz, both near harmonics will coincide and the ear will say that this is a consonance. In fact, this makes a pure third.

An interval is experienced as pure if the ratio of the frequencies can be written as a quotient of two small integer numbers. If the ratio of the frequency of note X to the frequency of the note Y can be written as n/m, then the m-th harmonic of X is equal to the n-th harmonic of Y. That can be seen easily. To find the lowest identical harmonics of the two notes of an interval one has only to simplify the quotient of the frequencies and apply this rule.

Why pureness of some intervals can be heard better than of some other on a hurdy gurdy

Intervals on a hurdy gurdy are heard between a chanterelle and a bourdon, usually with more than one octave distance, in many cases even two or three octaves. Although there also may sound a chien or a mouche at a pitch between the other two, this does not interfere much with the arguments here: it only blends with the overtones of the bourdon.

If the frequency ratio is n/m and n or m is rather high, the corresponding harmonic will be rather weak and relatively more close to other harmonics, so it will be more difficult to hear impureness, at the same time making pureness of that interval less important. If n or m is rather low and the other not too hight, pureness can be heard easily. Examples:

1. Pureness of a fifth (3:2) can be heard easily, still better a fifth plus one octave (3:1), a little less so a fifth plus two octaves (6:1).

2. Pureness of a major third (5:4) can be heard well, but pureness of a third plus one or two octaves (5:2 or 5:1) can be heard much better. So a pure third in the scale is very important on the hurdy gurdy.
3. Pureness of a fourth (4:3) can be heard rather easily, a fourth plus an octave (8:3) is a little more difficult and a fourth plus two octaves still more so. Moreover, already for a fourth plus an octave, the third plus two octaves (5:1) on the lowest note against the octave of the upper note (16:3) makes this interval rather dissonant. So, in practice, the pureness of the fourth of the scale is less important on the hurdy gurdy then pureness of the fifth and the major third.

4. About the same can be said for the major sixth (5:3).

5. Pureness of a minor third (6:5) can be heard rather well, but with one or two octaves added (12:5 or even 24:5) it is very difficult; near overtones make it much more dissonant. So it is not important on a hurdy to have pure minor thirds.

6. The same is true for the minor sixth (8:5).

7. The pureness of the ninth (second plus octave) (9:4) can be heard rather much better than the second (9:8), and one octave higher (9:2) it is even more so. Therefore, it is nice to have pure seconds in the scale of a hurdy gurdy.

Conclusion: the order of priority of pureness in the scale of a hurdy gurdy from important to unimportant is as follows.

1. prime (with one or more octaves distance to the bourdon)
2. fifth
3. major third (easiest tuning with two or one octave added)
4. fourth (easiest tuning with no octaves added)
5. second/ninth (easiest tuning with two octaves added)
6. major sixth (easiest tuning with no octaves added)

(and rather unimportant: minor third, minor sixth, minor seventh, major seventh, and the remaining chromatics).

There is a strange pitfall in tuning a minor third-plus-octave(s): this must be almost 1/5th semi-tone larger than in equal temperature, but one might be tempted to make it too small, in fact a third semi-tone smaller than in equal temperament. In that case, it is tuned to a 7-th harmonic. For instance, if an es' is tuned to a c, one might get tempted to make the bes", as the third harmonic of es', equal to the 7-th harmonic of the c, which in fact does not belong to our tonal system, but yielding a fine blending interval, better than the pure minor third-plus-octave itself. However, making the interval larger, you will arrive to another blending interval where g''' is the corresponding 5-th harmonic of the es' and the 12-th harmonic of the c.

Pure tuning or not?
The experience of impureness of impure intervals depends on the instrument and the way it is used:

- If the instrument has a dark tone, impureness of major thirds will be noticed less clearly by the ear as the fifth harmonic is rather weak already. An instrument with a light and clear tone colored by many harmonics (not to be confused with the noise-component of the scratching of a wheel with too much resin) will have a much more audible harmonic on two octaves plus a third and then a scale with a pure major third will make the instrument sound much more beautiful.

- If you want to play together with other instruments you must agree more or less on the choice of temperament. A little difference may be acceptable. By putting a little wax into some fingerholes of the bagpipe of your mate it is rather easy to make his instrument play in the same fine scale as your hurdy gurdy.

- The beauty of pure intervals may be less clear in some complex modern music than in traditional or early music.

So you have to decide for yourself what you wish, what you can tolerate, what is unacceptable on your instrument, in your music and in playing together with others.

You might prefer for folk music or medieval music to play a pure harmonic scale, sacrificing the possibility of changing the bourdon.

On the contrary, you may like harsh dissonances because of the musical tension. For instance, in playing medieval (gothic) music (up to 1500) you might wish to use Pythagorean temperament; it is the authentic temperament for medieval music, drone music included, and it can be wonderful if you can enjoy the bold Pythagorean thirds. In standard Pythagorean tuning, changing bourdons to F / C / G / D / A / E / B is no problem.

Basic theoretical ideas for tuning with pure intervals: frequency ratio and the cent system

Intervals are made pure by making the frequency ratio's correct. For an octave this is 1:1, a fifth 3:2, a fourth 4:3, a major third 5:4, an minor third 6:5, a second 9:8 or 10:9. These ratios cause some harmonics of the two tones to coincide and this pleases the ear. For instance, if the a' is tuned to 440 Hz, its fifth is 3/2x440 Hz = 660 Hz.

By shortening the vibrating length of a melody string with the tangents one gets higher notes. The frequency is inverse linear to the vibrating length. For
instance, if the chanterelle is shortened to 2/3 of its length, the frequency is multiplied with 3/2, making it sound the fifth of the open string tone.

By an array of four successive fifths, for instance F-C-G-D-A (or an array of a fifth above, a forth beneath, a fifth above, a fourth beneath) one reaches a third between the first and last tone (apart from octave differences). This third is called the Pythagorean third; it is very impure: its frequency ratio is \(3/2 \times 3/2 \times 3/2 \times 3/2 = 27/8\), while a pure third has ratio 3/2 = 80/64.

Instead of calculating with frequency ratio's, always multiplying and dividing, it is easier to calculate with cents. Here the logarithm is used. Don't fear log but use it as a tool, it is easy.

- A pure octave is a difference of 1200 cent
- A pure fifth is (about) 702 cent
- A fourth 498 cent
- A major third 386 cent
- A minor third 316 cent
- A second is 204 or 182 cent, but a ninth is heard as a pure interval if the difference is 204 + 1200 cent.

Now one has only to add and subtract in cent. For instance the pure third F-A makes a difference of 386 cent, the pure minor third A-C a difference of 316 cent, so the fifth F-C originating from stacking these two intervals has a difference of 386+316=702 cent and that makes a pure fifth. However, stacking 4 fifths F-C-G-D-A yields 702+702+702+702=2808 cent. That is 2*1200 +408 cent: two octaves plus 408 cent, and this 408 cent is much more than a pure third of 386 cent: the Pythagorean third is 22 cent (almost 1/4 of a semitone) larger than a pure third.

Calculating relative to equal temperament

Equal temperament is defined by making each semitone 100 cent. So here the fifth (seven semitones up) is 7 x 100 = 700 cent, not much less than the 702 cent of a pure fifth. But the third (4 semitones up) is 400 cent, much more than the 386 cent of a pure third, but also considerable less than the 408 cent of the Pythagorean third.

It is very easy to calculate in cents relative to equal temperament. In practice that means forgetting multiples of 100. Then a pure fifth (702 cent) is +2 cent, a fourth (498 cent) is -2 cent, a major third (386 cent) is -14 cent, a minor third is +16 cent. This system is used in the accompanying spreadsheet.
Here is an example: Suppose that we want the G as the first bourdon, the base of our system. The cent system is a relative system, so we can choose the G to be 0 cent. Now a D on the chanterelle will be a fifth plus some octaves higher. If we want this fifth to be pure, the D will be a multiple of 1200 cent plus 702 cent. However, we forget the multiples of 100, so the D is +2 cent relative to equal temperament. If we would want the Fis to be pure to D, it is a pure third up, so we can add -14 cent, so the Fis will be -12 cent relative to equal temperament. If we want now the Cis to be the pure fifth to this Fis, we should add +2 cent, so the Cis will be -10 cent relative to equal temperament. And if we want the C to be pure to G (=0 cent) then this is a pure fourth up to G, so we should add -2 cent to that 0 cent, making the C -2 cent relative to equal temperament.

Here is another example: If you see that the F is 4 cent relative to equal temperament and the A is -8 cent relative to equal temperament, then the third is (-8) - (4) = -12 cent relative. The pure third is -14 cent relative, so this third is almost a pure third but not quite so.

If you might want the mathematical definition: the cent difference is defined by 1200 times the 2-log of the frequency ratio.

The proposed tuning explained

Personally I love the natural harmonic approach. I wish all fifths, major thirds, fourths and seconds plus one or more octaves between chanterelle and bourdons to be pure. Now I will suppose that there are a bourdon in C and one in G. Then there is a choice to make for the A on the chanterelle. I prefer a pure ninth over a pure sixth-plus-octave, so I tolerate an A that is impure to a C bourdon in favor of the A being pure to the G bourdon (and the D-chien!). The consequence is the tuning system set out in a previous section: a practical proposal.

By demanding fifths to be pure to the tones of the bourdons and chiens in C, G and D, one must make C-G-D-A pure. The major thirds on C, G and D define E, B and Fis.

The other tones are not defined by strict demands on pureness of intervals. The F may be defined as a fourth (plus octave(s)) to the C. The Bes, Es and As/Gis may be defined as sixths to the drones. The Cis may be defined as a third to the A. The result is a type of temperament that might have been rather usual in the 15th century: a modified Pythagorean system that yields some pure thirds.

In this system you can play in C (only the sixth, the A, is 20 cent sharp to a pure sixth to C, but with a distance of at least one octave extra, that is not important) and in G. Both are fine and make all essential intervals pure.

If you want to play in D as well, the E is a quarter of a semitone flat; if you can accept that sometimes, when necessary, is up to you.
If you want to play in F, the third F-A is a quarter of a semitone sharp, it is a really harsh (Pythagorean) third; so F should not be used unless you like the effect of a dissonant third.

*These restrictions are the logical unavoidable consequences of the mentioned wishes on pureness.*

On the hurdy gurdy the tunings of Cis, Es and As are not critical and if you don't want to play in D or F even Fis and Bes are not critical for the experience of pureness. So you may want to change them. The accompanying spreadsheet might be helpful to see the effects of changes.

Here is a table for this system. The second column indicates the basic way of the tuning of each note in principle, the third and fourth column present the frequency ratio's to the open g and the distance in cent. The fifth column presents a possible modification of the system in order to come nearer to equal temperature without sacrificing the experience of pure intervals. The following two columns present the vibrating part of the string and the calculated distance of the corresponding tangent to the nut if you would play without pressure on the key. The same is done for equal temperature in the following two columns and for the modified pure system in the last two columns. The column "dist.diff. to wheel" presents the difference in positions for the corresponding tangent between the preferred tuning and equal temperature.

This table is created with a [downloadable spreadsheet](#) in the Open Document Format. and can be used with [Open Office](#) or with [Google spreadsheets](#).

"Open Document Format" is a modern, open, general standard format.

Open Office is a complete, easy, office suite that can read all usual formats of the past, including from MSOffice. It is available for free on all modern operating systems.

```
<table>
<thead>
<tr>
<th>key</th>
<th>definition</th>
<th>rel.freq</th>
<th>cent-rel</th>
<th>modific</th>
<th>preferred rel.freq</th>
<th>pure temp rel.length</th>
<th>dist 345</th>
<th>equal temp rel.length</th>
<th>dist 345</th>
<th>dist.diff. to wheel</th>
<th>modified rel.length</th>
<th>dist 345</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>1</td>
<td>1.00</td>
<td>0</td>
<td></td>
<td>1.00</td>
<td>0.0</td>
<td>1.00</td>
<td>0.0</td>
<td>1.00</td>
<td>0.0</td>
<td>1.00</td>
<td>0.0</td>
</tr>
<tr>
<td>gis</td>
<td>3rd under C</td>
<td>1.07</td>
<td>12</td>
<td>0</td>
<td>0.938</td>
<td>21.6</td>
<td>0.944</td>
<td>19.4</td>
<td>2.2</td>
<td>0.944</td>
<td>19.4</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>4th under D</td>
<td>1.13</td>
<td>4</td>
<td></td>
<td>0.889</td>
<td>38.3</td>
<td>0.891</td>
<td>37.6</td>
<td>0.7</td>
<td>0.889</td>
<td>38.3</td>
<td></td>
</tr>
<tr>
<td>bes</td>
<td>3rd under D</td>
<td>1.20</td>
<td>16</td>
<td>12</td>
<td>0.833</td>
<td>57.5</td>
<td>0.841</td>
<td>54.9</td>
<td>2.6</td>
<td>0.835</td>
<td>56.9</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>3rd on G</td>
<td>1.25</td>
<td>-14</td>
<td>-6</td>
<td>0.800</td>
<td>69.0</td>
<td>0.794</td>
<td>71.2</td>
<td>-2.2</td>
<td>0.800</td>
<td>69.0</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>4th on G</td>
<td>1.33</td>
<td>-2</td>
<td></td>
<td>0.750</td>
<td>86.3</td>
<td>0.749</td>
<td>86.5</td>
<td>-0.3</td>
<td>0.750</td>
<td>86.3</td>
<td></td>
</tr>
<tr>
<td>cis</td>
<td>3rd on A</td>
<td>1.41</td>
<td>-10</td>
<td>-6</td>
<td>0.711</td>
<td>99.7</td>
<td>0.707</td>
<td>101.0</td>
<td>-1.4</td>
<td>0.710</td>
<td>100.2</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>5th on G</td>
<td>1.50</td>
<td>2</td>
<td></td>
<td>0.667</td>
<td>115.0</td>
<td>0.667</td>
<td>114.7</td>
<td>0.3</td>
<td>0.667</td>
<td>115.0</td>
<td></td>
</tr>
<tr>
<td>es</td>
<td>3rd under G</td>
<td>1.60</td>
<td>14</td>
<td>6</td>
<td>0.625</td>
<td>129.4</td>
<td>0.630</td>
<td>127.7</td>
<td>1.7</td>
<td>0.628</td>
<td>128.4</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>3rd on C</td>
<td>1.67</td>
<td>-16</td>
<td></td>
<td>0.600</td>
<td>138.0</td>
<td>0.595</td>
<td>139.9</td>
<td>-1.9</td>
<td>0.600</td>
<td>138.0</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>4th on C</td>
<td>1.78</td>
<td>-4</td>
<td></td>
<td>0.563</td>
<td>150.9</td>
<td>0.561</td>
<td>151.4</td>
<td>-0.4</td>
<td>0.563</td>
<td>150.9</td>
<td></td>
</tr>
</tbody>
</table>
```
With strings in a standard Bourbonnais lay out, you have chanterelles in D and bourdons in D and G, possibly with a chien or mouche in G/D/A. In this case, all in the above is to be transposed a fourth lower and you will be able to play in G and D, if necessary in A and only with a Pythagoreen third C-E in C. The same positions of the tangents as in the previous setting can be used.

That is not the case when you would choose to tune the chanterelle to C and want to play in C and G, but the previous ideas can be used as well because the two keys to be used differ a fourth or a fifth.

A renaissance or medieval hurdy gurdy (Henri III or gothic model or a symphonie) with open chanterelles in G can benefit from tuning the bourdons (or one bourdon with a capo) to A and D, facilitating especially Dorian and Mixolydian scales. Then the same ideas can be applied, but with all tones transposed one tone higher. If the chanterelles can be tuned to C, you may tune the bourdons to D and G and transpose the ideas a fourth lower.

Using elementary electronic tuners as a tuning aid

Apart from some rare types all electronic tuners use equal temperament. These can be used as a tuning aid by using the indicator of the deviation or the ability to change the A form 440 Hz to something higher or lower. If changes can be made in 1 Hz steps only, this is a little crude. You can use the table in the above for calculations.

In stead of a stand-alone electronic tuner you might want to use a software electronic tuner. Many of them are available, some of them to be bought, others for free.
If you want to calculate by hand, there is a simple rule for differences smaller than 40 cent. If the A (around 440 Hz) is set 1 Hz higher, all tones are made (almost) 4 cent higher. So, if you first want to tune the A to 440Hz, then want to make the Cis pure to that A, so making the Cis 14 cent lower than it would be in equal temperament, you can tune this with an electronic tuner set to 440 - 14/4 = 436.5 Hz (and 436 or 437 Hz are usable). Don't use this rule for too large differences: 1200 cent (one octave) higher you don't get 440 + 1200/4 = 740 Hz but 880 Hz.